

Indian Statistical Institute, Bangalore

B. Math (II)

First semester 2014-2015

Mid-Semester Examination : Statistics (I)

Date: 11-09-2014

Maximum Score 50

Duration: 3 Hours

1. How would you make sense of the statement, "we can get *binomial distribution* as a limit of *hypergeometric distribution*"?

[8]

2. Let X_1, X_2, \dots, X_n be a random sample from the distribution with *pdf* given by

$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1} I_{(\theta_1, \theta_2)}(x); \theta_1 < \theta_2 \in \mathbb{R}$$

Find *maximum likelihood estimators (MLE)* for θ_1 and θ_2 . Find the mean of the distribution and obtain *method of moments (MOM)* estimator for it.

[6 + (2 + 4) = 12]

3. If X_1, X_2 are independent *Gamma*(a, λ) and *Gamma*(b, λ) random variables; λ, a and b all positive, then find the distribution of $Y = \frac{X_1}{X_2}$. Based on Y_1, Y_2, \dots, Y_n ; a random sample from the distribution of Y , find *method of moments (MOM) estimators* for λ, a and b . If $\lambda > 0$ and a, b positive integers; were all known then explain how you would generate observations on Y .

[8 + (2 + 3 + 3) + 6 = 22]

4. Explain how you would generate observations, using a coin, from uniform distribution on $\{1, 2, \dots, N\}$, where N is a known positive integer.

[10]

5. Let X_1, X_2, \dots, X_n be a random sample from the *Poisson*(λ), $\lambda > 0$ distribution. Find (*MLE*) for λ when *a*) there is no nonzero observation, *b*) there is at least one nonzero observation.

[3 + 5 = 8]